

# CONFORMALLY SYMMETRIC CONTRIBUTIONS TO BFKL EVOLUTION AT NEXT TO LEADING ORDER

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## Abstract

Unitarity corrections to the BFKL evolution at next to leading order determine a new component of the evolution kernel which is shown to possess conformal invariance properties. Expressions for the complete spectrum of the new component and the correction to the intercept of the pomeron trajectory are presented.

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The program of resummation of the leading ( $\sim \alpha_s \text{Log } 1/x$ )<sup>n</sup> and next-to-leading order corrections to the small-x behaviour of the gluon structure functions has undergone a revival of interest, mainly motivated by the new experimental results on DIS at HERA, which seem to be related to pomeron exchange. The small-x region of QCD, however, merges with the *softer* “Regge limit of QCD” at small transverse momenta, and it is this second limit which can be better described by reggeon unitarity in the *t*-channel[1]. Summation of the next to leading logs ( $\sim [c_1\alpha_s + c_2\alpha_s^2] \text{Log } 1/x$ )<sup>n</sup> in the Regge limit has special features connected to the phenomenon of reggeization of the gluon. Reggeization can be proven in perturbation theory and its effects can be “iterated” in the construction of an evolution kernel using perturbative s-channel unitarity, as done by Lipatov and collaborators and by Bartels[1]. Our results are derived by imposing conditions of *t*-channel unitarity on the scattering amplitude in the 2-reggeon sector. A new scale invariant component of the BFKL evolution is identified, which is expressed in terms of massless transverse momentum diagrams [1]. The new component is given by

$$\frac{1}{(g^2 N)^2} K^{(4n)} = \left( \mathcal{K}_0 \right) + \left( \mathcal{K}_1 \right) - \left( \mathcal{K}_2 \right), \quad (1)$$

where

$$\mathcal{K}_0 = \frac{1}{8\pi} \eta^2 (k^2)^D \left( \delta^2(k - k') + \delta^2(k + k') \right) \quad (2)$$

$$\begin{aligned} \mathcal{K}_1 = & \frac{\eta}{8\pi^2} \left( 2k^2 k'^2 [(k' - k)^2]^{D/2-2} + [(k + k')^2]^{D/2-2} \right. \\ & \left. - (k^2 [k'^2]^{D/2} + [k^2]^{D/2} k'^2) \left( \frac{1}{(k' - k)^2} + \frac{1}{(k' + k)^2} \right) \right) \end{aligned} \quad (3)$$

and

$$\mathcal{K}_2 = \frac{\eta}{4\pi^2} \frac{k^2 k'^2 (k^2 - k'^2)}{(k + k')^2 (k - k')^2} \left( (k^2)^{D/2-1} - (k'^2)^{D/2-1} \right). \quad (4)$$

We have set  $D = 2 + \epsilon$  and  $\eta = 2/(D - 2)$ .  $\left( \mathcal{K}_0 \right) + \left( \mathcal{K}_1 \right)$  can be reexpressed in terms of the square of the lowest order Lipatov kernel.  $\left( \mathcal{K}_2 \right)$  is a new component which is separately infrared safe and has distinct conformal invariance properties.

The complete eigenvalue spectrum of  $K^{(4n)}$  is given by  $N^2 g^4 \mathcal{E}(\nu, n)$  where

$$\mathcal{E}(\nu, n) = \frac{1}{\pi} [\chi(\nu, n)]^2 - \Lambda(\nu, n). \quad (5)$$

$\chi(\nu, n)$  is the Lipatov characteristic function giving the  $O(g^2)$  eigenvalues.  $\Lambda(\nu, n)$  is due to the new  $\mathcal{K}_2$  component and is given by

$$\Lambda(\nu, n) = - \frac{1}{4\pi} \left( \beta' \left( \frac{|n| + 1}{2} + i\nu \right) + \beta' \left( \frac{|n| + 1}{2} - i\nu \right) \right). \quad (6)$$

where

$$\beta'(x) = \frac{1}{4} \left( \psi' \left( \frac{x+1}{2} \right) - \psi' \left( \frac{x}{2} \right) \right), \quad (7)$$

with

$$\psi'(z) = \sum_{r=0}^{\infty} \frac{1}{(r+z)^2}, \quad (8)$$

from which it follows that the eigenvalues  $\Lambda(\nu, n)$  are all real. We can also write [1]

$$\begin{aligned} \Lambda(\nu, n) &= -\frac{1}{8\pi} \left( \beta'(m) + \beta'(1-m) + \beta'(1-\tilde{m}) + \beta'(\tilde{m}) \right) \\ &\equiv \mathcal{G}[m(1-m)] + \mathcal{G}[\tilde{m}(1-\tilde{m})] \end{aligned} \quad (9)$$

where  $m = 1/2 + i\nu + n/2$  and  $\tilde{m} = 1/2 + i\nu - n/2$ . This is the holomorphic factorization property satisfied by the leading order eigenvalues  $\chi(\nu, n)$ , which is directly related to conformal invariance.

Finally we discuss the numerical values that we obtain from our results. The leading eigenvalue is at  $\nu = n = 0$ , as it is for the  $O(g^2)$  kernel. From  $\mathcal{K}_2$  alone, the correction to the Pomeron intercept  $\alpha_0$  is (we recall that  $\alpha_0 - 1$  gives the inverse power behavior of  $F_2(x)$ )

$$\frac{9g^4}{16\pi^3} \Lambda(0, 0) \sim -16.3 \frac{\alpha_s^2}{\pi^2} \quad (10)$$

The complete  $\hat{K}^{(4n)}$  gives

$$\begin{aligned} \mathcal{E}(0, 0)/(16\pi^3) &\sim \frac{N^2 g^4}{16\pi^4} \left( [2\ln 2]^2 - 1.81 \right) \\ &\sim \frac{9g^4}{16\pi^4} \times 0.11 \sim \frac{\alpha_s^2}{\pi^2} \end{aligned} \quad (11)$$

giving a very small positive effect. However, the disconnected parts of  $K^{(4n)}$  do not have a consistent reggeization interpretation[1]. To obtain a consistent scale-invariant  $O(g^4)$  kernel it is necessary to subtract the square of the leading-order kernel. This gives, as a modification of  $\alpha_0$ ,

$$\begin{aligned} \frac{\tilde{\mathcal{E}}(0, 0)}{16\pi^3} &= \frac{N^2 g^4}{16\pi^4} \left( -3[\chi(0, 0)]^2 - \Lambda(0, 0) \right) \\ &\sim \frac{9g^4}{16\pi^4} \times (-5.76 - 1.81) \\ &\sim -68 \frac{\alpha_s^2}{\pi^2} \end{aligned} \quad (12)$$

which is a substantial negative correction. The scale-invariant kernel should accurately describe the full next-to-leading-order kernel in the transverse momentum infra-red region. We conclude that this region can produce a strong reduction of the BFKL small-x behavior.

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### References

- [1] For more details and all references see C. Corianò and A. R. White ANL-HEP-PR-94-84, to appear in *Phys. Rev. Lett.*, and ANL-HEP-PR-95-12 (hep-ph 9503 294).